NUMERICAL SOLUTION OF THE SPECTRAL RADIATION—GASDYNAMIC PROBLEM OF RADIATIVE COOLING OF A SPHERICAL PLASMA VOLUME TAKING INTO ACCOUNT NONSTATIONARY RADIATION TRANSFER

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UDC 533.6.011.8:535.2

The method of averaging is generalized to the case of the nonstationary equation of radiation transfer taking into account scattering. Results are presented for calculations of radiative cooling of a spherical plasma volume.

In high-temperature gasdynamic problems with temperatures on the order of several kilo-electron-volts, the radiation energy becomes comparable to the energy in matter. In this case, energy transfer must be described by a nonstationary equation [1]. Methods for solving nonstationary transfer equations were examined in [2, 3] for the case of spherical symmetry in the approximation of a gray gas without taking into account scattering with the simplest angular dependence of the radiation intensity. Methods for solving such an equation, taking into account both angular as well as the frequency dependence of the radiation intensity have been developed only for the stationary transfer equation without taking into account scattering [4-9]. It is of interest to solve the more general problem, which is free of the limitations indicated.

In solving the problem of radiative cooling of a spherical plasma volume expanding into a vacuum, we propose a generalization of I. V. Nemchinov's method of averaging the transfer equation [7] to the case of the nonstationary radiative transfer equation, for a medium with both absorption and scattering. (This method was successfully applied in a number of works [10-12] to problems of low-temperature radiative gasdynamics taking into account continuum and selective absorption.) The system of equations of radiative gasdynamics taking into account radiation energy and pressure is solved with the help of a completely conservative difference scheme. A number of variants of the problem are calculated with the use of both the approximation of a totally ionized gas and with tabulated values of the thermodynamic and optical gas parameters.

1. The equation of radiative transfer in an absorbing and scattering medium in spherical coordinates is given by

$$\frac{1}{c}\frac{\partial J_\varepsilon}{\partial t}+\frac{1}{r^2}\frac{\partial (r^2\mu J_\varepsilon)}{\partial r}+\frac{1}{r}\frac{\partial[(1-\mu^2)J_\varepsilon]}{\partial \mu}=\varkappa_\varepsilon' B_\varepsilon-(\varkappa_\varepsilon'+\varkappa_s)J_\varepsilon+\frac{\varkappa_s}{2}\int_{-1}^{1}p(\mu,\ \mu')J_\varepsilon(\mu')\,d\mu';$$

$$\mu=\cos\Theta;\ B_\varepsilon=\frac{15}{\pi^4}\sigma\varepsilon^3\left[\exp\left(\frac{\varepsilon}{T}\right)-1\right]^{-1}. \tag{1}$$

Here and in what follows, having in view a plasma with temperatures on the order of kilovolts, we will take into account only classical scattering of photons by electrons, for which the scattering indicatrix [13] is

$$p(\mu,\ \mu')=\frac{3}{8}[3-\mu^2+(3\mu^2-1)\mu'^2]. \tag{2}$$

Following [7], we set:

$$F_\varepsilon^+=\int_0^1 J_\varepsilon d\mu;\quad F_\varepsilon^-=\int_{-1}^0 J_\varepsilon d\mu;\quad \psi_\varepsilon^\pm=J_\varepsilon/F_\varepsilon^\pm;\quad F_\varepsilon=F_\varepsilon^++F_\varepsilon^-;$$

A. N. Sevchenko Scientific-Research Institute of Applied Physical Problems. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 41, No. 2, pp. 318-327, August, 1981. Original article submitted June 30, 1980.

 0022-0841/81/4102-0906$07.50

TABLE 1. Initial Data and Parameters of the Problem for Variants Computed with Thermodynamic and Optical Properties of the Plasma Given in Different Forms

Parameters	Number of variant			
	1 (table)	2 (bremsstrahlung)	3 (table)	4 bremsstrahlung)
R_0, cm	1,0	1,0	0,1	0,1
T_0, keV	2,0	2,0	2,0	2,0
E_0, MJ	2370	1870	2,05	1,72
E_{u0}/E_0	0,189	0,098	0,062	0,020
λ_s	0,517	0,517	0,0517	0,0517
μ_0	194,9	194,9	194,9	194,9
$\bar{c}$	951,7	951,7	951,7	951,7
α	0,649	0,805	0,720	0,780

$$S_\varepsilon^\pm = 2c_\varepsilon^\pm F_\varepsilon^\pm; \quad c_\varepsilon^+ = \int_0^1 \mu\psi_\varepsilon^+ d\mu; \quad c_\varepsilon^- = \int_{-1}^0 \mu\psi_\varepsilon^- d\mu;$$
$$S_\varepsilon = S_\varepsilon^+ + S_\varepsilon^-; \quad U_\varepsilon = \frac{2}{c}(F_\varepsilon^+ + F_\varepsilon^-); \quad g_\varepsilon^\pm = \mp (J_\varepsilon)_{\mu=0}/(F_\varepsilon^+ + F_\varepsilon^-);$$
$$d_\varepsilon^+ = \int_0^1 \mu^2\psi_\varepsilon^+ d\mu; \quad d_\varepsilon^- = \int_{-1}^0 \mu^2\psi_\varepsilon^- d\mu. \tag{3}$$

Here, $g_\varepsilon^\pm$ is the sphericity factor defined, in contrast to [7], in such a way that it is possible to transform to the two-flux approximation [2].

In order to obtain the angle-averaged spectral transfer equation, it is necessary to integrate Eq. (1) taking into account (2) and (3) with respect to μ over the limits −1 to 0 and 0 to 1. In order to perform the frequency average, we separate the entire spectral interval into M frequency groups and, following [7], we introduce the average group quantities:

$$F_i^\pm = \int_{\varepsilon_1}^{\varepsilon_2} F_\varepsilon^\pm d\varepsilon; \quad (\varphi_\varepsilon^\pm)_i = F_\varepsilon^\pm / F_i^\pm; \quad S_i^\pm = 2c_i^\pm F_i^\pm; \quad c_i^\pm = \int_{\varepsilon_1}^{\varepsilon_2} c_\varepsilon^\pm (\varphi_\varepsilon^\pm)_i \, d\varepsilon;$$
$$B_i = \int_{\varepsilon_1}^{\varepsilon_2} B_\varepsilon d\varepsilon; \quad S = \sum_{i=1}^M S_i = 2\sum_{i=1}^M (c_i^+ F_i^+ + c_i^- F_i^-); \quad U = \sum_{i=1}^M U_i =$$
$$= \frac{2}{c}\sum_{i=1}^M (F_i^+ + F_i^-); \quad d_i^\pm = \int_{\varepsilon_1}^{\varepsilon_2} d_\varepsilon^\pm (\varphi_\varepsilon^\pm)_i \, d\varepsilon; \quad \varkappa_i^\pm = \int_{\varepsilon_1}^{\varepsilon_2} \varkappa_\varepsilon' (\varphi_\varepsilon^\pm)_i d\varepsilon;$$
$$\varkappa_i^p = \frac{\int_{\varepsilon_1}^{\varepsilon_2} \varkappa_\varepsilon' B_\varepsilon d\varepsilon}{B_i}; \quad g_i^\pm = \frac{\int_{\varepsilon_1}^{\varepsilon_2} g_\varepsilon^\pm (F_\varepsilon^+ + F_\varepsilon^-)\, d\varepsilon}{F_i^+ + F_i^-}. \tag{4}$$

Averaging (1) with respect to angles and integrating the equation obtained with respect to frequency over the ranges of the i-th group, using (4), and introducing the distortion function for the spectrum $\eta_i^\pm = \varkappa_i^\pm / \varkappa_i^p$, we write the multigroup system of the radiative transfer equations:

$$\frac{1}{c}\frac{\partial F_i^\pm}{\partial t} + \frac{1}{r^2}\frac{\partial (r^2 c_i^\pm F_i^\pm)}{\partial r} + \frac{g_i^\pm}{r}(F_i^+ + F_i^-) = \varkappa_i^p (B_i - \eta_i^\pm F_i^\pm) \mp \frac{\varkappa_s}{2}(F_i^+ - F_i^-). \tag{5}$$

System (5) is exact in the sense that if at each point the true values of the averaged coefficients $c_i^\pm$, $g_i^\pm$, and $\eta_i^\pm$ are used (they can be obtained by solving the initial equation (1) and averaging according to the equations indicated above), then we obtain from Eqs. (5) the same values of $F_i^\pm$ obtained by solving the initial equation (1) with subsequent integration over angles and frequency. The advantage of the given method consists of the fact that, as experience shows, the averaging procedure can be carried out over a quite large number of time steps, retaining the values of the averaged coefficients in the

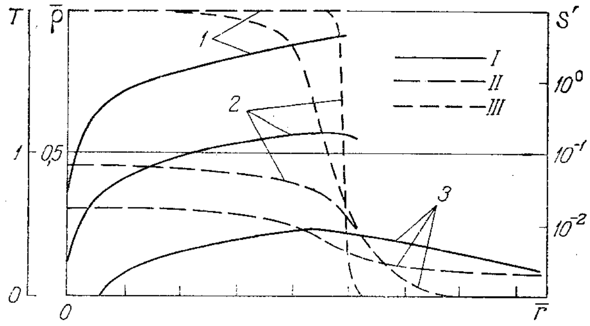

Fig. 1. Plasma temperature T, the dimensionless plasma density $\bar{\rho}$, the energy flux density of the radiation S' for variant 3 as a function of $\bar{r}$ at different times: I) $S' = \mu_0 \bar{S}$; II) T, keV; III) ρ; 1) $t_1 = 0$ sec; 2) $t_2 = 10^{-10}$ sec; 3) $t_3 = 10^{-9}$ sec.

intervals between the averaged values with respect to particular physical variables (mass, optical thickness, temperature, etc.), depending on the type of problem being solved. It may be assumed that since the averaged coefficients are integral functionals of J_ε, varying weakly with a change in the magnitude of J_ε, and mainly determined by the optical thickness of the region, in order to obtain the averaged coefficients, it is possible to use the stationary analog of Eq. (1), rather than Eq. (1) itself. As computational experience has shown, this is justified in view of the large difference in the scales of the characteristic times over which the radiation intensity and averaged coefficients vary.

2. As an example of the use of the method described above, we will examine the problem of radiative cooling of a spherical volume of dense plasma expanding into a vacuum with initial density ρ_0, radius R_0, and temperature T_0 (on the order of several kiloelectron-volts).

We write the total system of radiation—gasdynamic equations for the problem formulated as follows:

$$\frac{\partial u}{\partial t} + r^2 \frac{\partial}{\partial m}[p + p_R] = 0; \quad u = \frac{\partial r}{\partial t}; \quad v = \frac{1}{3}\frac{\partial r^3}{\partial m};$$

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial m}[(p + p_R) ur^2 + r^2 S] = 0; \quad E = \varepsilon_T + \frac{u^2}{2} + vU;$$

$$\frac{1}{c}\frac{\partial F_i^\pm}{\partial t} + \frac{1}{r^2}\frac{\partial (r^2 c_i^\pm F_i^\pm)}{\partial r} + \frac{g_i^\pm}{r}(F_i^+ + F_i^-) = \varkappa_i^p (B_i - \eta_i^\pm F_i^\pm) \mp \frac{\varkappa_s}{2}(F_i^+ - F_i^-); \tag{6}$$

$$p = p(\varepsilon_T, v); \quad U = \frac{2}{c}\sum_{i=1}^{M}(F_i^+ + F_i^-); \quad S = 2\sum_{i=1}^{M}(c_i^+ F_i^+ + c_i^- F_i^-);$$

$$p_R = \frac{2}{c}\int_0^\infty d\varepsilon \int_{-1}^{1} \mu^2 J_\varepsilon d\mu = \frac{2}{c}\sum_{i=1}^{M}(d_i^+ F_i^+ + d_i^- F_i^-).$$

The boundary and initial conditions are:

$$m = 0: F_i^+ = F_i^-; \quad J_\varepsilon(\mu) = J_\varepsilon(-\mu)_{|\mu>0}; \quad u = 0;$$

$$m = M_0/4\pi: F_i^- = 0; \quad J_\varepsilon(\mu)_{|\mu<0} = 0; \tag{7}$$

$$t = 0: u = 0; \quad p = p_0; \quad v = 1/\rho_0; \quad T = T_0; \quad F_i^\pm = F_{i\,st}^\pm;$$

$F_{st}^\pm$ are obtained from a solution of the system of stationary transfer equations, corresponding to (5).

Using as scale factors the quantities $T_* = T_0$; $r_* = R_0$; $v_* = 1/\rho_0$; p_*; $\varepsilon_{*1} = T_*$; $F_{\varepsilon *} = \sigma T_*^3$; $m_* = \rho_0 r_*^3$; $u_* = \sqrt{p_* v_*}$; $\varepsilon_{T*} = u_*^2$, and $t_* = r_*/u_*$, we put the system of equations (6) and (7) into dimensionless form, and in so doing, the problem will be characterized by the following parameters:

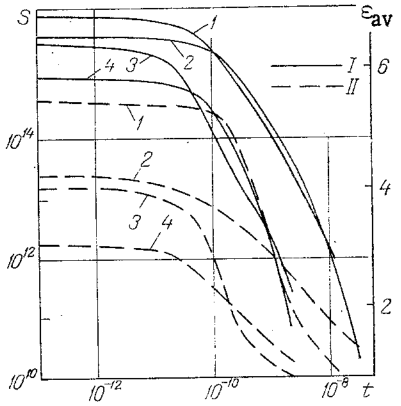

Fig. 2. Dependence on time t (sec) of the radiation energy flux density S (MJ/m²·sec) and ε_{av} (keV) at the plasma boundary for all variants: I) S; II) ε_{av}.

$$\mu_0 = \frac{\sigma T_*^4}{p_* u_*}; \quad \bar{c} = \frac{c}{u_*}; \quad \lambda_s = \varkappa_{s0} r_*; \quad \varkappa_{s0} = 0.39772\frac{z}{A}\rho_0. \tag{8}$$

3. Several types of methods exist for solving the stationary transfer equation, corresponding to (1). We used one of the variants of the S_n method. This choice stems from the fact that, first, the method is simply realized and, second, it is comparatively easy to take into account scattering in this case. Due to the cumbersomeness of the difference scheme, it is not given here. Its form is close to the scheme presented in [14]. In order to approximate the scattering integral, we used Gaussian quadrature equations for the intervals (−1, −0) and (0, −1).

In order to solve the transfer equations (5), we will use the absolutely stable, implicit, two-point iteration scheme with second-order accuracy in time and space, developed previously [2]. We note that, as shown in [15], for transfer equations, second-order difference schemes lead to radiative thermal conductivity in the limit of large optical thicknesses. This can also be demonstrated for the scheme used here.

Gaussian quadrature equations were used for finding the averaged coefficients in integrating with respect to angle. They were used also in integrating with respect to frequency in the region where the absorption coefficient has a smooth behavior. Trapezoidal rules were used for integrating over spectral lines, as well as near jumps in the absorption coefficient.

In order to write down the equations of gasdynamics in difference form, we will use the well-known "cross" scheme [16]. (The mass coordinate is divided into N − 1 intervals with step $h_{j+1/2} = m_{j+1} - m_j$):

$$\bar{u}_{j+1}^{n+1} = u_{j+1}^{n} - R_{j+1}^{n+1/2}\left[(p+p_R)_{j+3/2}^{n+1/2} - (p+p_R)_{j+1/2}^{n+1/2}\right]\frac{2\tau}{h_{j+3/2}+h_{j+1/2}};$$

$$\omega_{j+1/2}^{s+1} = \begin{cases} \mu_{00}\rho_{j+1/2}\delta u_{j+1/2}^2, & \delta u_{j+1/2} < 0 \\ 0 \quad , & \delta u_{j+1/2} \geqslant 0 \end{cases};$$

$$u_{j+1}^{s+1} = \bar{u}_{j+1}^{n+1} - R_{j+1}^{n+1/2}\left[\omega_{j+3/2}^{n+1/2} - \omega_{j+1/2}^{n+1/2}\right]\frac{2\tau}{h_{j+3/2}+h_{j+1/2}};$$

$$r_{j+1}^{s+1} = r_{j+1}^{n} + u_{j+1}^{n+1/2}\tau; \quad v_{j+1/2}^{s+1} = \frac{(r^3)_{j+1}^{s+1} - (r^3)_{j}^{s+1}}{h_{j+1/2}};$$

$$E_{j+1/2}^{s+1} = E_{j+1/2}^{n} - \left\{\left[(\bar{p}+p_R)uR + \mu_0 r^2 S\right]_{j+1}^{n+1/2} - \left[(\bar{p}+p_R)uR + \mu_0 r^2 S\right]_{j}^{n+1/2}\right\}\frac{\tau}{h_{j+1/2}};$$

$$\varepsilon_{\tau j+1/2}^{s+1} = E_{j+1/2}^{s+1} - 0.25\left[(u^2)_{j+1}^{s+1} + (u^2)_{j}^{s+1}\right] - \mu_0 v_{j+1/2}^{s+1} U_{j+1/2}^{s+1};$$

$$\tau = t^{n+1} - t^n; \quad \bar{p} = p + \omega; \quad p = p(\varepsilon_\tau, v). \tag{9}$$

Here,

$$F_{j+1/2} = 0.5(F_{j+1} + F_j); \quad \bar{F}^{n+1/2} = 0.5(\bar{F}^{s+1} + \bar{F}^n);$$

$$\tilde{F}_{j+1} = \frac{\tilde{F}_{j+3/2}h_{j+1/2} + \tilde{F}_{j+1/2}h_{j+3/2}}{h_{j+3/2}+h_{j+1/2}};$$

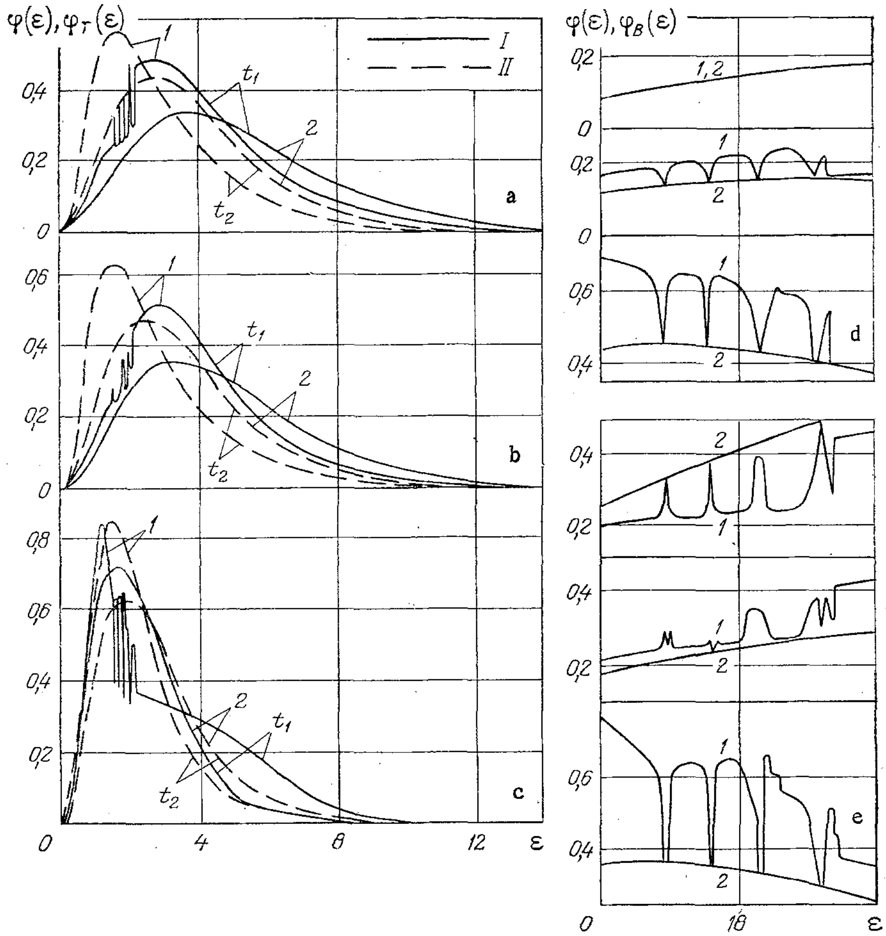

Fig. 3. Spectral characteristics of the outgoing radiation: a, b, and c) dependence on ε (keV); 1) $\varphi(\varepsilon)$; 2) $\varphi_T(\varepsilon)$; I) variant 3; II) variant 4; a) $t_1 = 0$ sec, $t_2 = 0$ sec; b) $t_1 = 10^{-11}$ sec, $t_2 = 2.08 \cdot 10^{-11}$ sec, c) $t_1 = 9.88 \cdot 10^{-11}$ sec, $t_2 = 1.19 \cdot 10^{-10}$ sec); d and e illustrate the dependence on ε; 1) $\varphi(\varepsilon)$; 2) $\varphi_B(\varepsilon)$ for variants 1 and 3; d) $t_1 = 0$ sec, $t_2 = 1.46 \cdot 10^{-10}$ sec, $t_3 = 8.71 \cdot 10^{-10}$ sec; e) $t_1 = 0$ sec, $t_2 = 10^{-11}$ sec, $t_3 = 9.88 \cdot 10^{-11}$ sec).

$$R^{n+1/2} = \frac{1}{3}\left[(r^2)^{s+1} + r^{s+1}r^n + (r^2)^n\right]; \quad (r^2)^{n+1/2} = 0{,}5\left[(r^2)^{s+1} + (r^2)^n\right];$$

$$\tilde{F} = \{p, \bar{p}\}; \quad F = \{u, R, r, S, U\}; \quad \bar{F} = \{u, R, r, S, U, p, \omega\}.$$

The system of equations written above is implicit and has second-order accuracy both in space and time. In order to solve it, an iteration procedure is used. As in [17], it can be shown that in solving transfer equations according to second-order schemes, the system of difference equations written down will be completely conservative.

We will now list the successive stages in the solution of the entire problem. Assume that at some time total averaging has been carried out, i.e. the true values of the averaged coefficients $c_i^{\pm}$, $g_i^{\pm}$, $\eta_i^{\pm}$, and $d_i^{\pm}$ have been found. Then, for some number of time steps, the total problem, in which the averaged coefficients are retained at the mass points, is solved. At each time step, an iteration procedure is used until the flux $F_i^{\pm}$ converges with relative error ε_{00}. After some number of time steps, the averaging procedure is repeated.

4. The calculations were carried out for radiative cooling of an aluminum plasma sphere, expanding into a vacuum, with initial temperature $T_0 = 2$ keV and initial radii $R_0 = 0.1$ cm and $R_0 = 1$ cm. For each initial size, the calculation was carried out 1) in the approximation of total ionization of the plasma with bremsstrahlung absorption coefficients

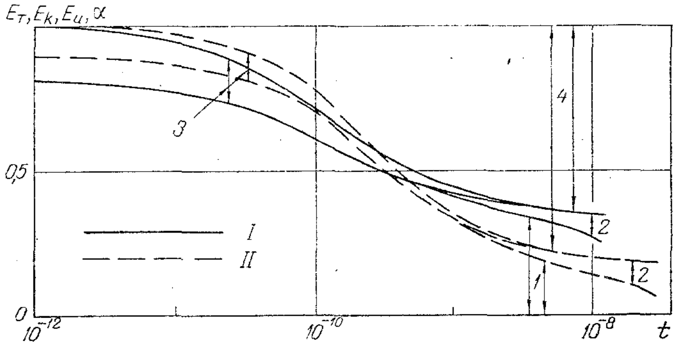

Fig. 4. Dependence on time t(sec) of 1) E_T, 2) E_K, and 3) E_R, referred to E_0, and 4) α for variants 1 and 2: I is variant 1 and II is variant 2.

(variants 2, 4); and, 2) with tabulated equation of state and absorption coefficients, taking into account spectral lines [18] (variance 1 and 3).

The calculations for all variants were carried out with seven spectral groups, determined by the following boundaries: 0.1388-0.2512-1.259-2.073-5.012-10.0-25.12-39.81 keV. The frequency average was carried out for 56 frequencies in the case of bremsstrahlung absorption coefficients and for 200 frequencies in the case of tabulated absorption coefficients, which permitted taking into account such properties of the actual spectrum as jumps in the absorption coefficient and spectral lines. In averaging over the angle, four angles were chosen in each hemisphere.

Let us examine the results of the computations. Table 1 shows the basic characteristics of the variants chosen.

Calculating the variants with different initial radii permitted clarifying the dependence of the process of radiative cooling on the initial optical thickness. Comparison of the bremsstrahlung variant and the variant using real absorption coefficients showed how much the computational results differ both qualitatively and quantitatively. Let us first consider the effect of gasdynamic parameters on the plasma cooling process. It is evident from Fig. 1 that up to the time $t \sim 10^{-10}$ sec for $R_0 = 0.1$ cm and $t \sim 10^{-9}$ sec for $R_0 = 1.0$ cm, the process of radiative cooling with an almost stationary plasma predominates. At later times, intense expansion leads to strong rarefaction of the gas mass near the boundary, radiative cooling slows down, and the plasma temperature equalizes.

We will now examine the change in the spectral characteristics of the radiation with time. Figure 2 shows the time dependence of the energy fluxes S at the plasma boundary and the photon energies ε_{av} averaged over the spectrum for all variants:

$$\varepsilon_{av} = \frac{\int_0^\infty \varepsilon U_\varepsilon d\varepsilon}{\int_0^\infty U_\varepsilon d\varepsilon}.$$

The brightness T_b and effective T_{eff} temperatures can be compared: $T_b = (S/\sigma)^{1/4}$, $T_{eff} = \varepsilon_{av}/3.832$.

It is evident from Fig. 2 that the qualitative behavior of the fluxes in variants 1 and 2 and, correspondingly, in variants 3 and 4 is more uniform than the behavior of the magnitudes of ε_{av}, reflecting the spectral characteristics of the radiation.

Figure 3 illustrates the functions characterizing the spectral composition of the outgoing radiation:

$$\varphi(\varepsilon) = \frac{\bar{S}_\varepsilon}{\bar{S}}; \quad \varphi_T(\varepsilon) = \frac{\bar{B}_\varepsilon(T_b)}{\bar{S}}$$

for a number of times in variants 3 and 4. The qualitative behavior of these quantities in variants 1 and 2 is the same (with the exception of the time scale of the evolution of the spectrum), so that they are not presented. The role of lines in the formation of the spectrum can be seen from Fig. 3, and it is also possible to get an idea of the degree of departure from equilibrium in the outgoing radiation. The region of the spectrum, in which the lines play a noticeable role, is shown in greater detail in Fig. 3d for variants 1 and 3, and in e for variant 3; here, φ_B is the equilibrium spectrum, corresponding to the boundary temperature of the plasma.

From the behavior of the basic integral energy characteristics of the problem (Fig. 4), as well as the magnitudes of the fluxes S and ε_{av} (Fig. 2), the considerable qualitative and quantitative difference between the computational results for the bremsstrahlung variant and the variant using the real absorption coefficients is evident. We note that for variants 3 and 4 the corresponding curves in Fig. 4 are shifted to the left approximately by an order of magnitude in time.

The results presented show the basic laws governing the process of radiative cooling of a plasma cluster expanding into a vacuum. We also note that the results of calculations of similar variants of the problem for $\varkappa_s R_0 \geq 1$ that do not take into account scattering (as in [2]), which are not presented here, reveal the considerable influence of this process on the rate of deexcitation and on the resulting values of the quantity α.

NOTATION

J, radiation intensity increased by a factor of π in the calculation for a unit interval of photon energy; r, Euler coordinate; c, velocity of light; t, time; Θ, angle of intersection of the ray with the radius vector; $\varkappa_\varepsilon^!$, spectral absorption coefficient, corrected for induced emission; ε, photon energy; $\varkappa_s$, scattering coefficient; $p(\mu, \mu')$, scattering indicatrix; $\sigma = 1.0302 \cdot 10^3$ MW/eV$^4 \cdot$m^2, Stefan—Boltzmann constant; T, temperature; $F^\pm$, and F, average single-sided and total radiation intensities, increased by a factor of π; $c^\pm$ and $d^\pm$, average single-sided cosines and squares of cosines in the hemispheres; $S^\pm$, and S, single-sided and total flux densities of the radiation energy; U, radiation energy density; ε_1 and ε_2, lower and upper boundaries of the i-th spectral group; u, mass velocity; $m = \int \rho r^2 dr$, mass coordinate; p, pressure; ρ, density; v, specific volume; E, total specific energy; ε_T, specific internal energy; p_R, radiation pressure; E_T, E_K, and E_R, thermal and kinetic energies, and the radiation energy; $\alpha = 4\pi \int_0^t r_b^2 S dt / E_0$, fraction of energy radiated out; r_b, plasma boundary. The indices are as follows: 0, initial values of quantities; *, characteristic dimensional quantity; ε, photon energy; i, index of the spectral group; j, mass cell in the grid; n, number of the time step; s, iteration index; (—), symbol indicating the radiation fluxes and the Euler coordinate written in dimensionless form.

LITERATURE CITED

1. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High Temperature Hydrodynamic Phenomena, 2 Vols., Academic Press.
2. G. S. Romanov, B. N. Bazylev, and K. L. Stepanov, "Radiative cooling of a spherical cloud of completely ionized gas expanding into a vacuum," Dokl. Akad. Nauk BSSR, 22, No. 2, 138-141, (1978).
3. A. A. Charakhch'yan, "Numerical scheme for evolutionary problems with nonstationary radiation transfer," Zh. Vychisl. Mat. Mat. Fiz., 18, No. 2, 499-504 (1978).
4. V. Ya. Gol'din, "Quasidiffusion method for solving the kinetic equation," Zh. Vychisl. Mat. Mat. Fiz., 4, No. 6, 1078-1087 (1964).
5. V. Ya. Gol'din, G. V. Danilova, and B. N. Chetverushkin, "Approximate method for solving the nonstationary kinetic equation," in: Numerical Methods in Transport Theory [in Russian], Atomizdat, Moscow (1969), pp. 50-58.
6. D. S. Filippychev and B. N. Chetverushkin, "Method for averaging diffusion type equations with respect to photon energy," Zh. Vychisl. Mat. Mat. Fiz., 16, No. 6, 1601-1605 (1976).
7. I. V. Nemchinov, "Averaged radiation transfer equations and their use in solving gas-dynamic problems," Prikl. Mat. Mekh., 34, No. 4, 706-721 (1970).
8. Yu. D. Shmyglevskii, "Variant of the moment method for calculating the transport of selective radiation," Zh. Vychisl. Mat. Mat. Fiz., 17, No. 3, 785-790 (1977).
9. V. G. Sevast'yanenko, "Transfer of radiation in a real spectrum. Integration over frequency," Inzh.-Fiz. Zh., 36, No. 2, 218-230 (1979).

10. I. V. Nemchinov, V. V. Svetsov, and V. V. Shuvalov, "Solution of the problem of the propagation of strong, intensely radiating shock waves in air by averaging the equation of radiation transfer." Low-Temperature Plasma in Space and Ion Earth, Collection of Articles [in Russian], VAGO, Moscow (1977), pp. 47-89.
11. V. I. Bergel'son, T. V. Loseva, and I. V. Nemchinov, "Numerical calculation of the problem of propagation of a plane subsonic radiation wave in a gas by averaging the equation of radiation transfer," in: Numerical Methods in the Physics of Plasma [in Russian], Nauka, Moscow (1977), pp. 234-236.
12. I. V. Nemchinov and V. V. Svetsov, "Calculation of the development of a laser explosion in air taking into account radiation," Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 26-32 (1977).
13. S. Chandrasekar, Radiant Energy Transfer [Russian translation], IL, Moscow (1953).
14. P. N. Stevens, "Use of the discrete ordinates method in radiative shielding calculations," Nucl. Eng. Design, 13, 395-408 (1970).
15. B. C. Hankin, J. G. Hill, and A. G. P. Warham, "Numerical solution of the unsteady radiative transfer equation," J. Quant. Spectrosc. Radiat. Transfer, 11, 949-962 (1971).
16. R. D. Richtmyer and K. W. Morton, Difference Methods for Initial-Value Problems, Wiley (1967).
17. A. A. Samarskii and Yu. P. Popov, Difference Methods in Gas Dynamics [in Russian], Nauka, Moscow (1975).
18. G. S. Romanov, K. L. Stepanov, and S. I. Kas'kova, et al., "Spectral absorption coefficients of a high-temperature aluminum plasma," Otchet po NIR: Plasma [in Russian], NII PFP, Minsk, Gos. reg. No. B771439.

SOME EXACT SOLUTIONS TO EQUATIONS OF TRANSIENT FLOW WITH SUCTION FOR A VISCOUS FLUID

L. F. Kozlov and Yu. A. Ptukha

UDC 532.526

Self-adjoint asymptotic solutions to the equations of flow are constructed for a viscous fluid near a permeable plane boundary.

We consider the problem of transient plane flow of an incompressible power-law non-Newtonian fluid near an infinitely large permeable wall in the plane of the x axis (Fig. 1). The fluid is uniformly sucked through the wall at a velocity $V_0(t)$. At the instant of time $t = 0$ the wall is suddenly set in motion at a velocity $U_0(t)$ in the direction of the x axis [1].

We will consider only asymptotic solution, i.e., assume that all derivatives are $d/dx \equiv 0$. At infinity we let the velocity be not zero, as is usually done, but finite [2]. Under these assumptions, the equations of motion for a power-law fluid become

$$\frac{\partial v_1}{\partial t} + V_0(t)\frac{\partial v_1}{\partial y} = \frac{mn}{\rho}\left(\frac{\partial v_1}{\partial y}\right)^{n-1}\frac{\partial^2 v_1}{\partial y^2}, \tag{1}$$

$$\frac{dV_0}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{2}$$

with the boundary conditions for the components of velocity and pressure

$$v_1 = v_2 = 0 \text{ at } t = 0,\ y > 0, \tag{3}$$

$$v_1 = U_0(t),\ v_2 = V_0(t),\ p = p_0(t) \text{ at } y = 0,\ t > 0. \tag{4}$$

We will henceforth deal only with the case $|n| < 1$. From Eq. (2) and the boundary condition (4) we determine the pressure

Institute of Hydromechanics, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 41, No. 2, pp. 328-333, August, 1981. Original article submitted April 7, 1980.

